Math 5110 Introduction to Modern Analysis, Fall 2026

Basics

- Instructor: Luke Rogers
- Office hours: TTh 10-11am
- Classes: TTh 2:00pm to 3:15pm in MONT 414
- Course Description (from the catalog): Metric spaces, sequences and series, continuity, differentiation, the Riemann-Stielties integral, functions of several variables.
- Text: None required. Standard texts at roughly the level of this course that you might find useful are Rudin's Principles of Mathematical Analysis, Strichartz's The way of Analysis, Protter and Morrey's A first course in Analysis, and Tao's Analysis I and II (of which the material in the first 5 chapters of vol I will not be emphasized). Munkres' Analysis on manifolds is also good for the material on several variables. Each has strengths and weaknesses, and their material is not entirely overlapping. I recommend matching the relevant sections to our learning goals (below).
- Assessment: 30% for each of two Midterms, 40% Final. One midterm or the final should be taken as an oral exam.

Learning Goals By the end of the course you will definitely need to:

- Know the basic properties of the real numbers, including how existence of sup and inf distinguishes them from the naturals, integers and rationals, and how these notions relate to completeness of $\mathbb{R}$.
- Be able to work with equivalence relations and the concept of countability and uncountability.
- Understand basic notions in metric topology, especially those involving compactness, sequential compactness, completeness, total boundedness, and connectedness.
- Be able to work rigorously with sequences: boundedness, monotonicity, convergence, limit, limit points, subsequences, Bolzano-Weierstrass Theorem, and Heine-Borel Theorem.

- Be able to work rigorously with series: partial sums, convergence, absolute convergence, tests for convergence (comparison, ratio, root, alternating), geometric and p-series, partial summation, Abel's theorem.

- Be able to work rigorously with the formal definitions of limits and continuity of functions, including one-sided limits, infinite limits and limits at infinity.

- Know the properties of continuous functions on compact sets: image of compact set is compact, consequences for the existence of maxima and minima and uniform continuity; image of a connected set is connected, intermediate value theorem,

- Be able to work rigorously with the basic concepts of differential calculus on Euclidean spaces: definition of derivative, Rolle's Theorem, Mean Value Theorem, Taylors theorem, and their consequences.

- Be able to work rigorously with the basic concepts of the Riemann-Stieltjies integral calculus: Riemann sums, integrable functions, Fundamental theorem of Calculus in one and higher dimensions, Gauss-Green theorem.

- Understand the difference between pointwise and uniform convergence, the connection of uniform convergence to continuity and convergence of integrals, application to power series. Know the statements of the Arzela-Ascoli theorem and Stone-Weierstrass theorem and be able to use them in problems.

- Be able to use linear algebra for differential calculus in more than one dimension: give the definition of the derivative of a map between Euclidean spaces and compute it in problems, be able to use the concepts of rank and nulity and explain their geometric meaning for derivatives. Be able to state the implicit and inverse function theorems, apply them in problems and explain their intuitive content.

What is expected of you

My primary expectation is that you attempt to understand the material outside of class, especially by attempting to fill in the proof details I have (deliberately) omitted in my notes and solve the problems I suggest as exercises. My intent is that you working through the material in the notes

and watching the videos I provide will allow me to only lecture for part of the class time and for us to discuss problems in class that you have already thought about and worked on, so you are well-prepared to discuss them.

l suggest that this class should require a time commitment of at least 10 hours per week (including class time). For students who have limited undergraduate analysis, more time is likely to be needed. In addition to discussing material in class, most of you will also find it useful to have some sort of study group with whom you will discuss concepts from the text and homework problems.

I suggest that you attend class, at least most of the time. I will not check whether you attend, nor will there be required assessment items other than the midterms for which you need you to be in class, but there is honestly not much point in being in the course if you're not planning to show up.

Class time is where we'll try to make it easier and faster to get a fuller picture of the topic than you get from reading alone. In case it matters, I might call on you in class to explain what you understood the main points to be in the reading, or what you did on a problem, but your answer will not be assessed for course credit.

Assessment

I'm giving 30% for each of two midterms and 40% for the final. There will be the option to replace one or two exams with oral exams.

Practice problems

People sometimes want a list of problems that they can practice on for exams. Below are the problems from Rudin's POMA that I gave as homework exercises the last time I taught from that text.

- Ch 1 Exercises 1,2,4,5,7,8,12,13,14,15,17.
- Ch 2 Exercises 1,2,3,4,6,7,8,9,11,12,13,14,15,17,19,21,23,25.
- Ch 3 Exercises 1,3,6,8,9,11,14,16,19,21,22,23
- Ch 4 Exercises 2,3,4,5,6,7,8,9,10,11,13
- Ch 5 Exercises 2,4, 6,11,15,18,21,22,26
- Ch 6 Exercises 1,2,4,5,6, 8,9,10,11,12,15
- Ch 7 Exercises 3,4,5,7,9,11,12,13,16,19,21,22,24,25.
- Ch 9.Exercise 6,10,13,15, 16,20,25

Advice

I strongly encourage you to form study groups and/or discuss problems with your fellow students. One thing this does is normalizes the fact that sometimes the material is difficult for everyone. Another is that it helps you to be comfortable asking for more discussion of a topic in lecture. Finally, it helps you to solve problems.

I suggest that when you first look at a problem you try to solve it by yourself. Once you have your own solution, or have exhausted your efforts to find one, you are encouraged to discuss it with other students. Look at what at each person in the group has done, criticize it, try to find and fix the holes in each others proofs.

Don't be afraid to try a lot of different approaches and ideas in solving a problem. It is not unusual to go through 20 pages or more of scratch work to solve a problem. Hands-on work with the material is where the learning really happens, so make it count.

Academic Misconduct, Plagiarism, etc

Quite simply, you should never hand in work that is not your own – whether you copied from a friend or the internet or a book or whatever. If you struggled with a problem, looked somewhere for an idea, and used that idea, then you should make a note on your solution of where you got help. If you have any doubt about the acceptable standards for academic conduct, consult the university's community standards:
UConn Academic Integrity page

At this level of course I don't expect to see any plagiarism, so I hope this section is redundant. In undergraduate courses I usually penalize a first offence with a 10% loss of course grade and a second offence with an additional 30% loss of course grade; more than that is an automatic F and a referral to academic standards.

Non-Academic Support

The university has many support services for the stresses and difficulties encountered by students. I urge you to make use of these early – don't wait and let a small problem get bigger before you deal with it!. Among the many offices some of you may find useful are the Dean of Students, Student Health and Wellness (the Mental Health division may be particulary helpful if you are stressed) and the Center for Students with Disabilities. It is worth looking at what each of these offices do, so you know where to find help if you or a friend needs it. I should also mention the Office of Institutional Equity, which deals with matters of discrimination and harassment; I hope

you will not need their help, but it is good to be aware of the support they offer.

CETL links to policies, etc The university has policies covering many circumstances and issues that could arise in the classroom. The provost's office asks faculty to include a link to these in their syllabus.
Policy and procedure links for syllabi.

Use of AI/LLMs

Large language Models (aka AI) can produce very good mathematical arguments. Of course, you won't hand in any work produced by an LLM, but they can be quite useful as you study. For example, you are asked to complete the notes with proofs you write yourself. If you write a proof but are unsure if it is fully rigorous, one option is to ask an LLM to critique it – to tell you about any mathematical errors or omissions in your argument. If you have tried hard to prove something and feel stuck, you can tell the LLM the problem and what you have tried and ask for a hint as to how to proceed. If that doesn't get you unstuck, you could even ask for a sketch of the argument that should be used. These last should be done sparingly – you only learn through struggling with ideas, so you don't want to take too much of the struggle out of the work – but can be a good choice for problems where you are really unable to proceed. With that said, I'd prefer that you ask about these problems in class or in office hours whenever this is possible: the former helps other students, and the latter at least keeps me aware of what people are finding difficult about the material.

Course outline and rough schedule

I am putting together some notes and videos. The idea is that the notes will state results but give at most sketches of proofs. The videos will have discussion of concepts and I will draw pictures and give hints about how some results are proved. It is intended that you fill in the missing details of the notes to obtain a complete picture of the material. Using the notes and videos is also intended to free up some class time so I am not always lecturing but instead can give you time to work out steps yourself, work on material with each other, and answer your questions about the material.

It is vital that you look at the notes, watch the videos, and (most important) try to work through the details of the arguments before class, so that you are well prepared to discuss them in class!

- Week 1. Real numbers. Cardinality and countability. Much of this is likely to be review, but it is possible that there is some of it you haven't encountered and it is fairly likely there is material where you can't come up with the proofs immediately.

- Week 2. Metric space basics and metric topology. In the former the new material is likely to be the Cauchy-Schwarz inequality and Hölder inequality. The latter is the foundation of our work on limits; there are a lot of definitions, but the main issue is to get a good sense of the notion of open set and how to work with it.

- Week 3. Sequences and limits. Here we do abstract results about sequences, limits, and open sets. You have sequences in $\mathbb{R}^n$ in mind as examples, but we don't use any of the vector or field structure.

- Week 4. Sequences and series in Euclidean spaces. We link the vector structure to sequences and then connect the order structure of $\mathbb{R}$ to questions of convergence. In particular we see how the least upper bound property is related to metric completeness of $\mathbb{R}$. The results on series are mostly familiar from calculus.

- Week 5. Midterm 1 on 9/29. Compactness.

- Week 6. Connectedness. Perfect Sets. Continuous functions.

- Week 7. Continuity and compactness; uniform continuity. Continuity and connectedness. In the Euclidean setting these give the extreme value theorem and the intermediate value theorem.

- Week 8. Differential calculus: Rolle's theorem, Mean value theorem, Taylor's theorem.
- Week 9. Riemann integration: Riemann sums, integrability, Fundamental theorems of calculus in $\mathbb{R}$. Gauss-Green theorem.
- Week 10. Midterm 2 on 11/3. Banach fixed point theorem and consequences.
- Week 11. Pointwise and uniform convergence. Power series. Uniform convergence and Riemann integral.
- Week 12. Weierstrass approximation theorem and Arzela Ascoli theorem.
- Week 13. Thanksgiving break
- Week 14. Linear Analysis: Derivative, chain rule, higher derivatives.
- Week 15. Inverse and Implicit function theorems.
- Additional topics. I do not expect to be able to use class time to discuss these, but plan to have videos on at least the following: the construction of $\mathbb{R}$ via Dedekind cuts, the construction of the completion of a metric space, the Baire category theorem and its consequences.